

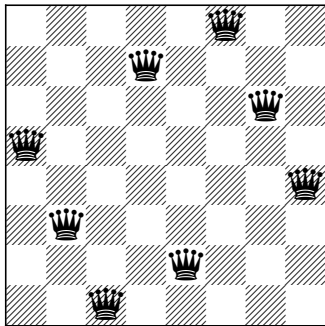
How Many Queens?

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N Queens Problem

- n queens on $n \times n$ chessboard
- no two queens on same row, column, or diagonal

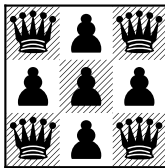


Maximum Queens Problem

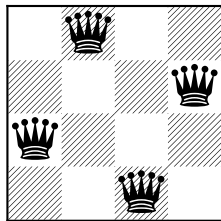
“What is the maximum number of [mutually nonattacking] queens one can place on an $n \times n$ board if one can block as many squares as he needs?” (Zhao, 1998)



1 on 2×2

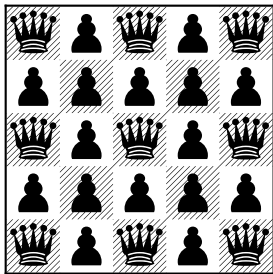


4 on 3×3

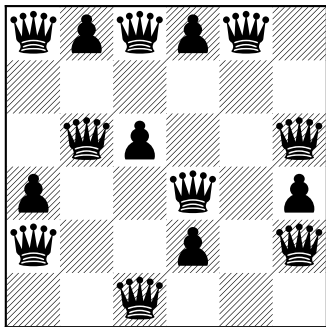


4 on 4×4

More solutions



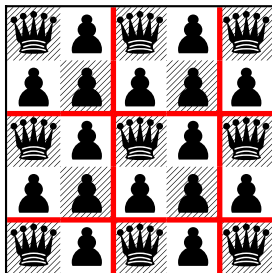
9 on 5×5



9 on 6×6

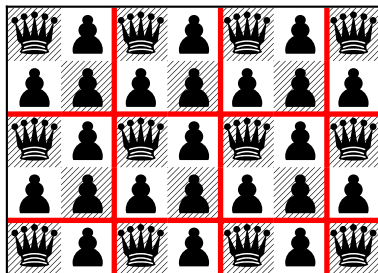
Answer

For an $n \times n$ board, the answer is $\lceil \frac{n}{2} \rceil^2$.



Answer for rectangular board

For an $m \times n$ board, the answer is $\left\lceil \frac{m}{2} \right\rceil \left\lceil \frac{n}{2} \right\rceil$.



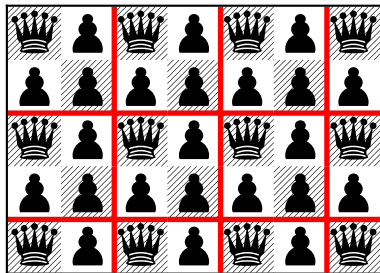
(*c.f.* kings independence number)

Min Pawns for Max Queens Problem

On an $m \times n$ board, how many squares do we need to block in order to place $M := \left\lceil \frac{m}{2} \right\rceil \left\lceil \frac{n}{2} \right\rceil$ mutually nonattacking queens on the board?

How many pawns? (m, n odd)

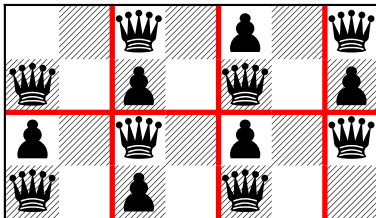
If both m and n odd, just one possible arrangement, with pawns in all squares unoccupied by queens.



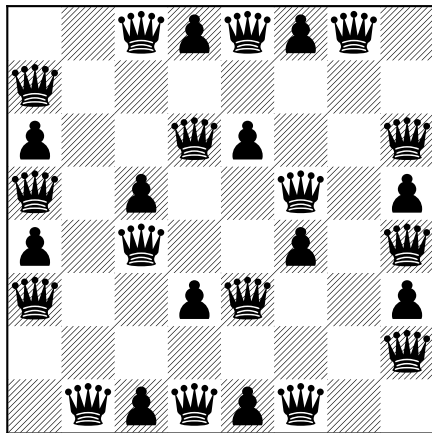
$$mn - M \text{ pawns}$$

How many pawns? (m or n even)

If m or n even, need at most $M - 2$ pawns.

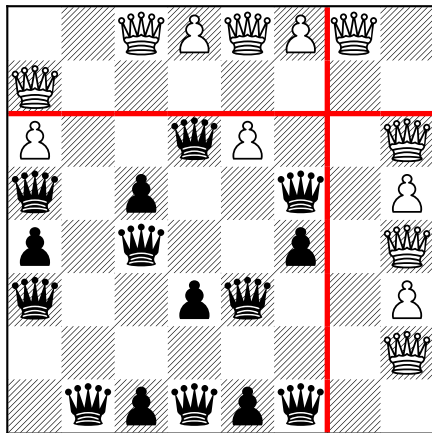


$$m = n = 4k, k \geq 1$$



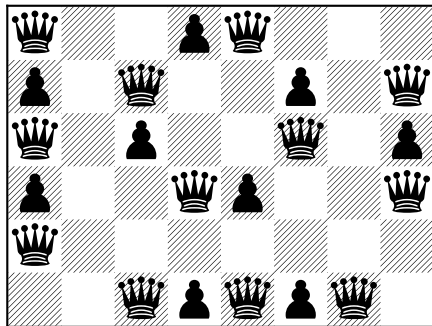
$M - 4$ pawns

$$m = n = 4k - 2, k \geq 1$$



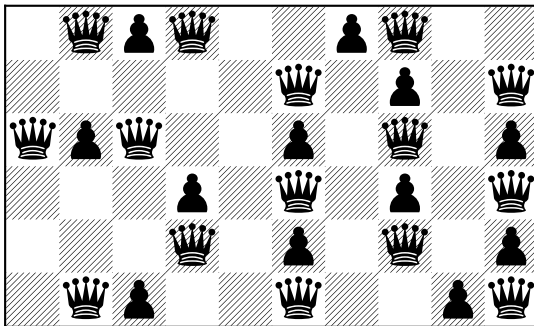
$M - 3$ pawns

6×8



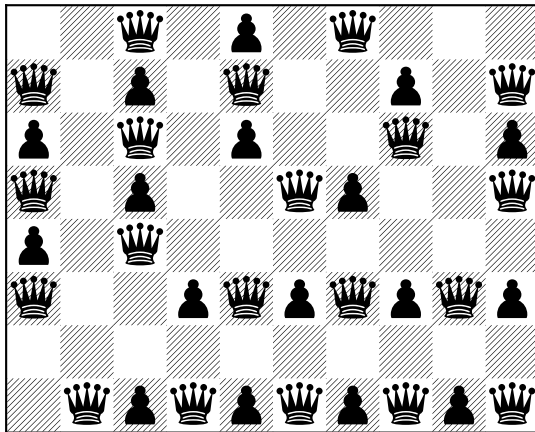
$M - 3$ pawns

6×10



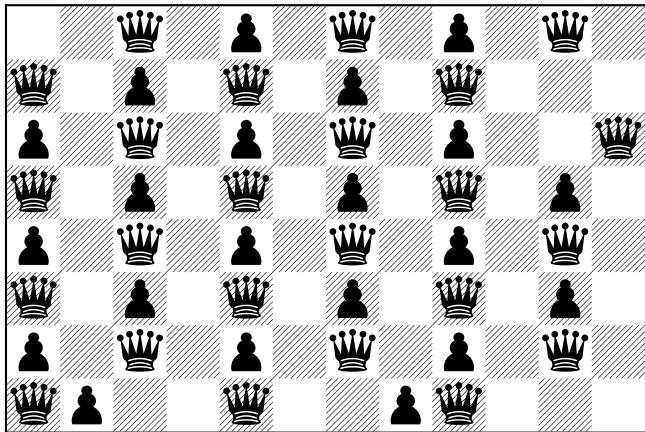
$M - 3$ pawns

8×10



$M - 3$ pawns

8×12



$M - 3$ pawns

Open Problems

- Minimum number of pawns for m or n even?
- Number of arrangements for m or n even?
- How many pawns are needed to place fewer than M queens?
- How about other types of board (e.g. torus)?

References

- Bell, J. & Stevens, B. (2009). A survey of known results and research areas for n -queens. *Discrete Math.* 309, no. 1, 1-31.
- Watkins, J. J., *Across the Board: The Mathematics of Chessboard Problems*, Princeton University Press, 2004.
- N+k Queens Problem Pages:
<http://npluskqueens.info>
- NEOS server at
<http://www.neos-server.org/neos/>

Any questions?